MGiaD: Multigrid in all dimensions. Efficiency and robustness by coarsening in resolution and channel dimensions

KI DELTA LEARNING
Scalable AI for Automated Driving

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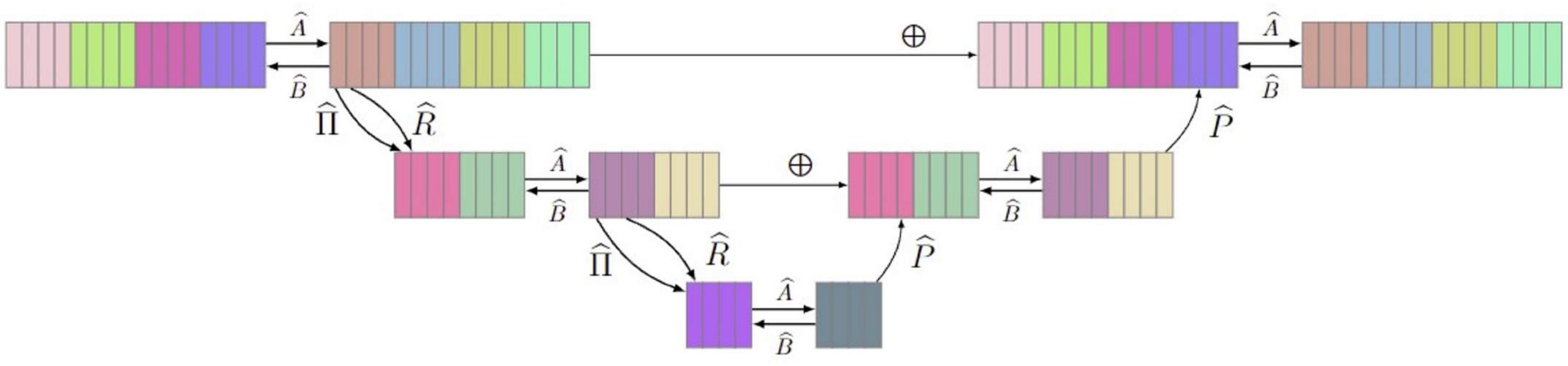

Figure 1: SiC-block (© University of Wuppertal)

Task

Reuse of modules and hierarchical structures in convolutional neural networks (CNNs) to achieve a more favorable **accuracy-parameter trade-off**.

- CNNs are build from 10-100 M of weights: risk of **overparameterization**
- Number of weights vs. accuracy
- How to increase the model capacity without requiring many weights?
- Exploit connection of multigrid methods (MG) and CNNs to get structured reduction

Approach

- Multigrid methods [1] are efficient hierarchical solvers for systems of (non-) linear equations
- CNNs and MG share the same properties [2]
- Exploit similarities for structured reduction [3]
- Sparsity in the resolution dimension: appropriate weight sharing, MgNet [2] (fig. 3)
- Sparsity in the channel dimension: Hierarchical structure, reduces weight count quadratic → linear (fig. 1)[4][3]

MG and CNNs

- MG are iterative methods to solve $Au = f$:

$$u^{i+1} = u^i + B\underbrace{(f - Au)}_{\text{residual}} \qquad i = 0,1,\ldots$$

- The (unknown) error propagation

$$e^{i+1} = \underbrace{(i - BA)}_{\text{ResNet-block}}e^i$$

- Resembles ResNet-block [5], yields motivation for reusing weights
- BUT: iterative methods are characterized by **slow convergence**
  → Restrict problem to coarser grid / resolution via pooling operations (Π, R)
- CNNs: data f and features u are related by (non-)linear mappings A, B

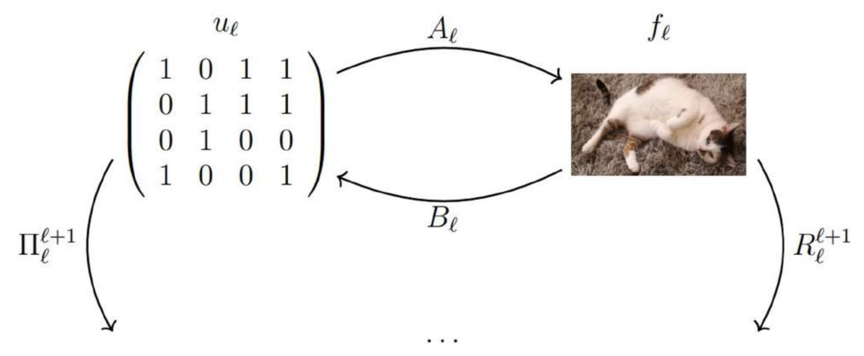

Figure 2: Data-feature mappings on resolution level l, followed by coarsening of data and residual to resolution level l + 1

- Iterative scheme on every resolution level as often as required (**smoothing**) (fig.2)
- Restrict residual $r_l^{l+1} = R_l^{l+1} r^l$
- Direct solving and correction of fine grid solution

References:

[1] Trottenberg , Osterlee and Schüller, Multigrid, 2001
[2] J. He and J. Xu, MgNet: A unified framework of multigrid and convolutional neural network, 2019
[3] van Betteray, Rottmann and Kahl, MGiaD: Multigrid in all Dimensions
[4] J. Ephrath et al. MGIC: Multigrid-in-Channels Neural Network Architectures, 2020
[5] Kaiming He et al., Deep Residual Learning for Image Recognition, 2015

Architecture

- A and B can be both shared, achieving a significant reduction (factor 4)
- Still the number of weights scale quadratically to the number of channels

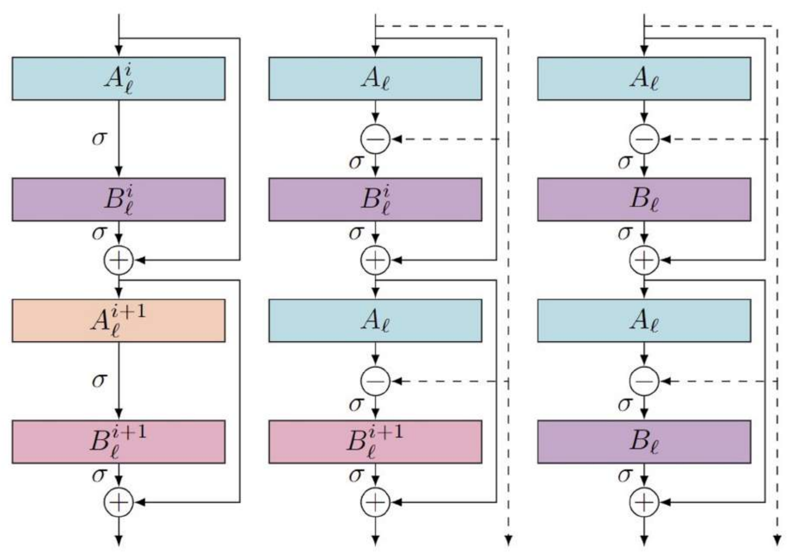

Figure 3: left: no weight sharing (ResNet); middle: sharing A, right: sharing both A and B (MgNet), each block corresponds to iteration step aka smoothing (© University of Wuppertal)

- Replacing A and B with grouped convolutions is not sufficient due to **lack of channel interaction**
  → Another hierarchical structure in the channel dimension build from grouped convolutions to **restore channel interaction**
- Number of channels is successively reduced until fully coupled convolutions are feasible
- Fully coupling corresponds to direct solving in multigrid
- On each channel level**: smoothing with shared weights (SiC)** (fig. 1)
- Successively increasing the number of channels, updating each level solution

Training Strategies

Results

Dataset	Model	λ	g_s	#weights(k)	acc. (%)
Cifar10	ResNet18	-	-	11, 174	95.58
	MgNet	-	-	2, 751	95.28
	MGiaD	1	8	139	90.82
	MGiaD	3	8	1, 269	**95.95**
TinyImageNet	ResNet18	-	-	11, 271	59.67
	MgNet	-	-	6, 396	59.48
	MGiaD	1	8	646	57.39
	MGiaD	1	64	2, 065	**60.17**

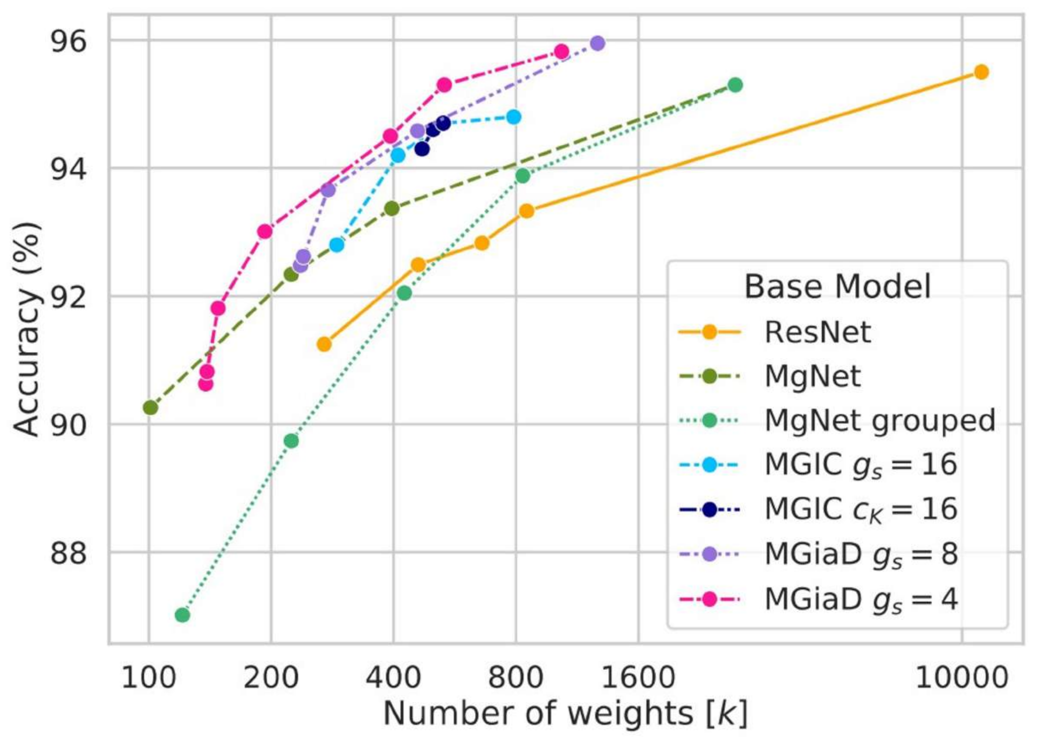

Figure 4: Models trained on Cifar-10, different group size, varying number on fully coupled channels, channel scaling

- Small group size: lesser weights
- High number of fully coupled channels: good accuracy

Partners

External partners

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